(1) When the numbers 1 through 64 are placed on the sixty-four squares of a checkerboard, define the gap to be the largest difference between two numbers on adjacent squares (two squares are adjacent if they share a side or even a corner). Determine the smallest possible gap among all ways of placing the numbers on the squares.

(2) Show that there are distinct positive integers $A, B_1, B_2, \ldots, B_{2007}$ such that each of the integers
\[ (B_1)^2 + A, \quad (B_2)^2 + A, \quad \ldots, \quad (B_{2007})^2 + A \]
is a perfect square.

(3) Let $\triangle ABC$ be an equilateral triangle with sides of length 1. If $n = 1, 2, \text{or} 3$, define $S_n$ to be the set of all points $D$ such that exactly $n$ of the angles $\angle ADB, \angle ADC$, and $\angle BDC$ are obtuse. Compute
\[ \text{area}(S_1) + 2 \text{area}(S_2) + 3 \text{area}(S_3). \]

(4) Suppose that $f(x, y)$ is a function of two real variables that satisfies
\[ f(x, a) + f(a, y) = f(x, b) + f(b, y) \]
for all real numbers $a, b, x,$ and $y$. Prove that there exists a function $g(x)$ of one real variable and a constant $C$ such that
\[ f(x, y) = g(x) - g(y) + C \]
for all real numbers $x$ and $y$.

(5) Suppose that $x$ is a real number such that $x + \frac{1}{x}$ is an integer. Prove that $x^n + 1/x^n$ is an integer for every integer $n$.

(6) Suppose you have four points in the plane with the property that any three of them can be simultaneously covered by a disk of radius 1. Show that all four points can be simultaneously covered by a disk of radius 1.

(7) Let $E$ be an ellipse of area 1 whose axes of symmetry are parallel to the $x$- and $y$-axes. Cut the interior of $E$ into four pieces using a horizontal line and a vertical line that intersect inside $E$. Prove that at least two of the resulting pieces have area at most $1/4$.

(8) Let $n$ be a positive integer, and let $e = 2.71828 \ldots$ denote the base of the natural logarithm. Prove that when $(n+1)!/e$ is rounded off to the nearest integer, the result is always a multiple of $n$. 