Introduction to Artificial Intelligence
COMP 3501 / COMP 4704-4
Lecture 14: Supervised Learning

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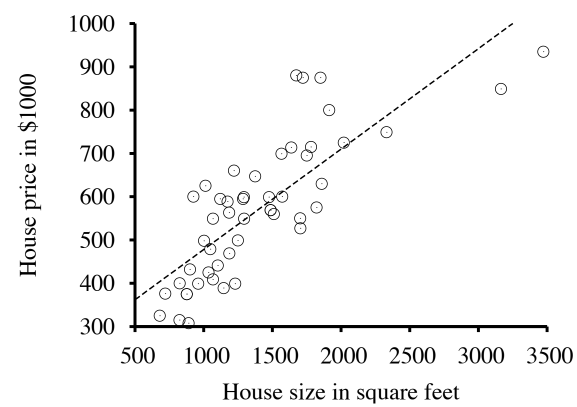
Today

- Supervised Learning
  - Linear regression (18.6)
  - Neural Networks (18.7)
- Making complex decision (17.1, 17.2, 17.3)
  - Background for Reinforcement Learning

Univariate linear regression

- Regression on a single variable
  - Input x, and output y; learn weights w
  - $y = w_0 + w_1 x$
- Hypothesis becomes
  - $h_w(x) = w_0 + w_1 x$
- Finding best weights is linear regression

Example

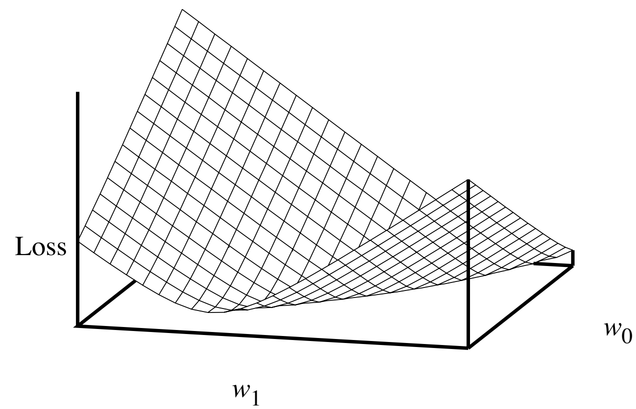


Learning

- Find the weights which minimize the loss
  - How do we define loss?
  - L_2 is the squared distance (to the line)
    - Also called the L_2 norm

$$Loss(h_w) = \sum_{j=1}^{N} L_2(y_j, h_w(x_j))$$
$$= \sum_{j=1}^{N} (y_j - h_w(x_j))^2$$
$$= \sum_{j=1}^{N} (y_j - (w_0 + w_1 x_j))^2$$

Graph of loss function



Learning

- In univariate case, can solve exactly
  - In general, use gradient descent to improve values
- Simple algorithm:
  - Initialize **w** randomly
  - Loop until converged:
    - For each wi in **w**
      - $w_i \leftarrow w_i - \alpha \, \partial/\partial w_i \, Loss(\mathbf{w})$

Gradient

$$Loss(h_w) = \sum_{j=1}^{N} (y_j - (w_0 + w_1 x_j))^2$$

- What is the gradient/slope with respect to w_0?
  - Just use a single training example, not all N
  - $\partial/\partial \mathbf{w}\, (y - h(x))^2 = 2(y - h(x)) \cdot \partial/\partial \mathbf{w}\, (y - h(x))$
  - $\partial/\partial w_0\, (y - h(x)) = \partial/\partial w_0\, (y - (w_0 + w_1 x)) = -1$
    - $\partial/\partial \mathbf{w}\, (y - h(x))^2 = -2 \cdot (y-h(x))$
  - $\partial/\partial w_1\, (y - h(x)) = \partial/\partial w_1\, (y - (w_0 + w_1 x)) = -x$
    - $\partial/\partial \mathbf{w}\, (y - h(x))^2 = -2 \cdot (y-h(x)) \cdot x$

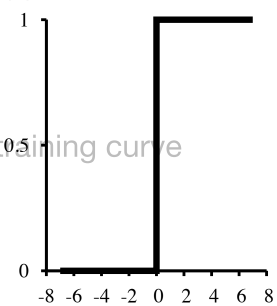
Final algorithm

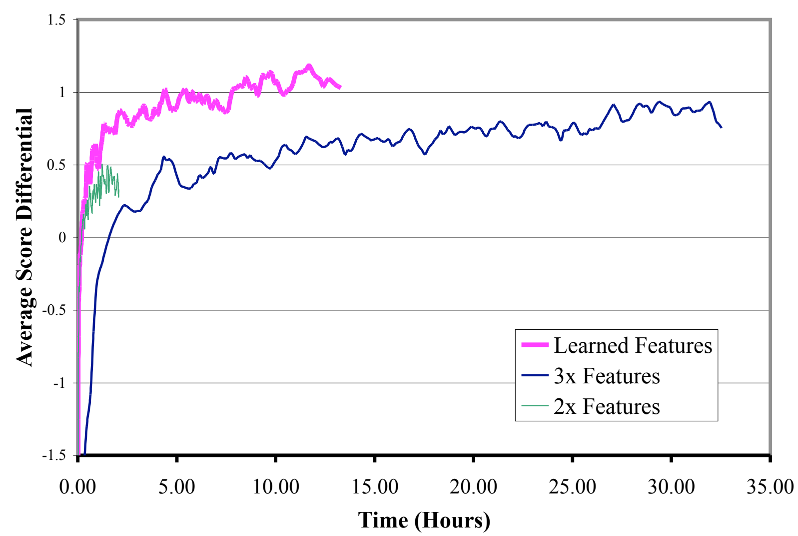
- Weight update was:
  - $w_i \leftarrow w_i - \alpha\, \partial/\partial w_i\, Loss(\mathbf{w})$
  - $w_0 \leftarrow w_0 + \alpha\, (y - h_w(x))$
  - $w_1 \leftarrow w_1 + \alpha\, (y - h_w(x)) \cdot x$
- For more variables:
  - $w_i \leftarrow w_i + \alpha\, x_i\, (y - h(x))$
  - Incremental update version
  - Batch update version in book

Example

Function vs classifier

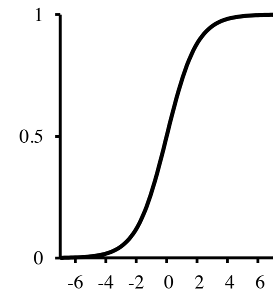
- So far we've learned a function approximator
  - Can also be used to learn a classifier
  - For instance, classify as true if $h(x) \geq 0$
    - Or $h_w(x) = 1$ if $w \cdot x \geq 0$
    - Or $h_w(x) = \text{Threshold}(w \cdot x)$
- Can visualize performance with a training curve



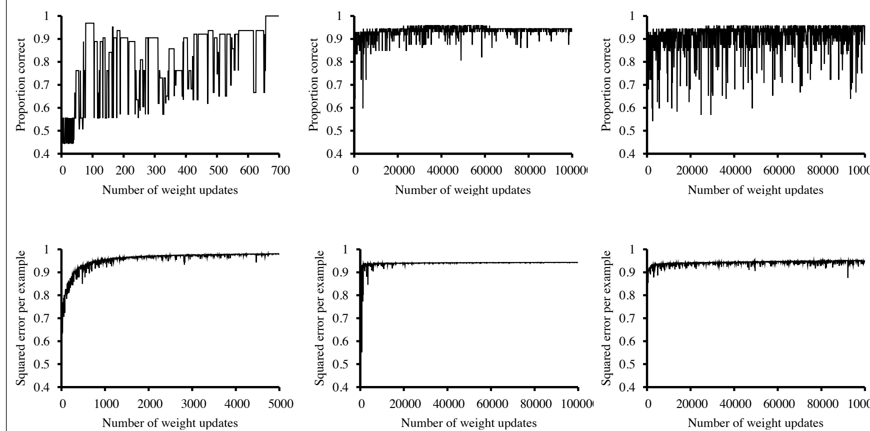


Threshold function

- What if we use a different threshold function
  - Get a different type of regression
- $Logistic(z) = \frac{1}{1+e^{-z}}$
- $h_w(x) = Logistic(w \cdot x)$
- Derivative of logistic:
  - $g'(w \cdot x) = g(w \cdot x)(1-g(w \cdot x))$
  - $w_i \leftarrow w_i + \alpha\,(y-h(x)) \cdot (h(x)(1-h(x))x_i$



Linear vs Logistic regression



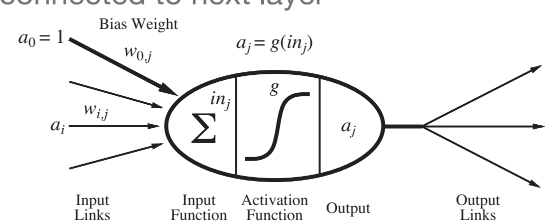
$\alpha = 1000/(1000+t)$

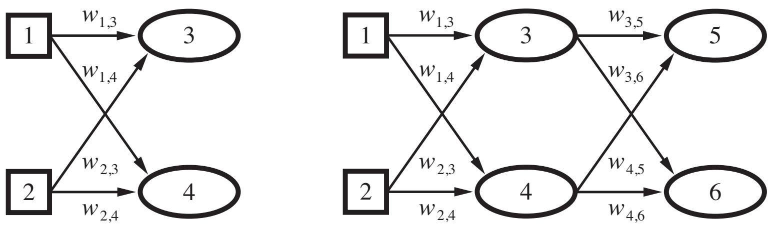
Notes

- Linear regression is limited to linearly separable classes
  - BUT, can use more complex inputs to make the classes linearly separable [kernel]
- Regression can easily learn majority function
  - Recall, this was hard for decision trees

Extending to multiple layers

- What we have so far is called a perceptron
  - Single-layer neural network
- Can extend to multiple layers
  - Usually assume that all outputs from previous layer fully connected to next layer





Neural Network training

- Use same rule as before to train the output layer
  - Let Err_k be the error in the *k*th output
    - Or assume a single output unit and ignore *k*
  - Let $\Delta_k = Err_k \cdot g'(in_k)$
  - Update rule becomes:
    - $w_{j,k} \leftarrow w_{j,k} + \alpha \cdot a_j \cdot \Delta_k$

Neural Network training

- Input rule more tricky
  - Where do the errors come from?
  - Let $\Delta_j = g'(in_j) \sum_k w_{j,k} \Delta_k$
  - $w_{i,j} \leftarrow w_{i,j} + \alpha \cdot a_i \cdot \Delta_j$
  - Full derivation in book