Worksheet 4 Answer Form–
You may choose to use this SN file to formulate your answers to Worksheet 4: 3.4 #2, 6, 10, 11, 19.
You should print it up and hand in a PAPER COPY to me at the beginning of lecture on Thursday, October 18.
Note that as with all worksheets, quizzes, and exams, you are expected to SHOW YOUR WORK. So whenever there is an answer blank, you should make sure to either type out what you did to get the answer, or attach a sheet with your “work” on it. Remember also that YOU DO NOT NEED TO SIMPLIFY YOUR ANSWERS.

(2.) The graph of a typical average cost function $A(x) = \frac{C(x)}{x}$, where $C(x)$ is a total cost function associated with the manufacture of $x$ units of a certain commodity is shown in the figure in the book.

a.) Explain in economic terms why $A(x)$ is large if $x$ is small and why $A(x)$ is large if $x$ is large.

Answer: When you first start producing a product you have startup costs that seem significant per item when you aren’t making many items. As you produce more items at a time those fixed costs divide out over the number produced and the average cost to you goes down. But once you hit a certain production capacity you’ll find that you need to put another chunk of money in–buy new machines or hire more employees, do more maintenance, etc.

b.) What is the significance of the numbers $x_0$ and $y_0$, the $x$- and $y$-coordinates of the lowest point on the graph of the function $A$?

Answer: The average cost is lowest ($y_0$) when you produce $x_0$ units so this is the point at which your production is most cost-effective (efficient).

(6.) MARGINAL AVERAGE COST
The management of ThermoMaster Company, whose Mexican subsidiary manufactures an indoor-outdoor thermometer, has estimated that the total weekly cost (in dollars) for producing $x$ thermometers is

$C(x) = 5000 + 2x$

a.) Find the average cost function.

$\bar{C}(x) = \frac{C(x)}{x} = \frac{5000 + 2x}{x} = \frac{1}{x}(2x + 5000) = \frac{5000}{x} + 2$

b.) Find the marginal average cost function

$\bar{C}’(x) = \frac{d}{dx} \left( \frac{5000}{x} + 2 \right) = -\frac{5000}{x^2}$

c.) Interpret your results.

Answer: Since $\bar{C}(x)$ is always negative, the average cost decreases as production increases. We can use the average cost function itself $C(x)$ to see that since

$\lim_{x \to \infty} C(x) = \lim_{x \to \infty} \left( \frac{5000}{x} + 2 \right) = 0 + 2 = 2$, the more you produce the closer your
average cost will be to $2 per item, the lowest possible average cost.

(10.) MARGINAL REVENUE
The management of Acrosonic plans to market the ElectroStat, an electrostatic speaker system. The marketing department has determined that the demand for these speakers is

\[ p = -0.04x + 800 \quad (0 \leq x \leq 20,000) \]

where \( p \) denotes the speaker’s unit price (in dollars) and \( x \) denotes the quantity demanded.
a.) Find the revenue function.
\[ R(x) = x(-0.04x + 800) = -x(0.04x - 800) \]

b.) Find the marginal revenue function.
\[ R'(x) = \frac{d}{dx}(-0.04x^2 + 600.0x - 3.0 \times 10^5) = 600.0 - 0.08x \]

c.) Compute \( R'(5000) \) and interpret your results.
\[ R'(5000) = 800 - 0.08(5000) = 400.0 \text{ dollars/unit} \]
So the approximate revenue you get from sale of the 5001st unit when the first 5000 have already been sold is $400. Notice we can compare that to the actual revenue realized from sale of unit 5001 by subtracting
\[ R(5001) - R(5000) = 5001(0.04(5001) - 800) + 5000(0.04(5000) - 800) = $399.96 \]

(11.) MARGINAL PROFIT
Refer to Exercise 10. Acrosonic’s production department estimates that the total cost (in dollars) incurred in manufacturing \( x \) ElectroStat speaker systems in the first year of production will be

\[ C(x) = 200x + 300000 \]
a.) Find the profit function.
\[ P(x) = -x(0.04x - 800) - (200x + 300000) = -200x + x(0.04x - 800) - 300000 = -0.04x^2 + 600.0x - 3.0 \times 10^5 \]
b.) Find the marginal profit function.
\[ P'(x) = \frac{d}{dx}(-0.04x^2 + 600.0x - 3.0 \times 10^5) = 600.0 - 0.08x \]
c.) Compute \( P'(5000) \) and \( P'(8000) \).
\[ P'(5000) = 600 - 0.08(5000) = 200.0 \text{ dollars/unit} \]
\[ P'(8000) = 600 - 0.08(8000) = -40.0 \text{ dollars/unit} \]
d.) Sketch the graph of the profit function and interpret your results.
Interpretation:
When we’ve sold 5000 units, the profit is increasing and the approximate profit realized from the sale of the 5001st unit would be $200 greater. Once we pass about 7500 units sold (the high point of the curve), the costs overwhelm again and the amount we profit starts to decrease.

(19.) MARGINAL PROPENSITY TO CONSUME
Suppose a certain economy’s consumption function is
\[ C(x) = 0.873x^{1.1} + 20.34 \]
where \( C(x) \) is the personal consumption expenditure and \( x \) is the personal income, both measured in billions of dollars. Find the rate of change of consumption with respect to income, \( \frac{dC}{dx} \). This quantity is called the marginal propensity to consume.
Answer: \( \frac{dC}{dx} = \frac{d}{dx} (0.873x^{1.1} + 20.34) = 0.9603x^{0.1} \)

Find the marginal propensity to consume when \( x = 10 \).
Answer: \( \frac{dC}{dx} (10) = 0.9603(10)^{0.1} = 1.2089 \) billion dollars of consumption per billion dollars of personal income