Homework submitted by the beginning of class on Oct 8 will be graded and available for pickup the afternoon of Wed Oct 9. The Midterm Exam will be held on Thursday Oct 10.

Start now!

The Problems

1. Page 50: \[11\]

In order to get some intuition about this problem, I suggest that you first work through the example below. You need only write the solution to problem 11, though.

**Example:** Fix \(K \in \mathbb{N}\). Let \(x\) be the sequence of \(K\) zeros followed by ones forever. That is, \(x_n = 0\) if \(n \leq K\) and \(x_n = 1\) if \(n > K\). So, the sequence \(x\), written in list form, appears as follows:

\[
\begin{array}{ccccccccc}
0 & 0 & \cdots & 0 & 1 & 1 & 1 & 1 & \cdots \\
\end{array}
\]

Clearly, \(x_n \to 1\) as \(n \to \infty\).

Define a new sequence \(z = (z_n)\) by \(z_n = \frac{1}{n} \sum_{j=1}^{n} x_j\). **Prove that** \(z_n \to 1\) as \(n \to \infty\).

2. Pages 54-5: 4

3. Pages 57-8: 3

4. Pages 61-3: 1

5. ![In this problem, you’ll explore \(\lim_{n \to \infty} A^{1/n}\) for \(A > 1\).](image)

(a) Prove that if \(A > 1\), then \(\lim_{n \to \infty} A^{1/n} = 1\).

**Big Hint:** Write \(A = (1 + x_n)^n\). (Notice that \(A^{1/n} = 1 + x_n\).) Now use problem 10 on problem set 1 to obtain an estimate on \(x_n\).

(b) Suppose that \(A = 1000001 = 10^6 + 1\). Based on your estimate in part (a), how large must \(n\) be so that \(A^{1/n} \leq 2\)?

(c) Using a calculator, find a much smaller \(n\) (than the one you found in part (b)) so that \(A^{1/n} \leq 2\).

Remarks: This shows that the estimate you used to prove part (a) is strong enough to obtain the desired result, but actually throws away lots of information. We’ll continue to use these types of estimates in the next problem set when I ask you to prove something much stronger, namely that \(\lim_{n \to \infty} n^{1/n} = 1\).