Introduction to Real Analysis
Math 3161 (Autumn 2013)
Problem Set 6

Due: Tuesday, Oct. 29.

As usual, start now! This problem set looks short because it takes up only 1/2 page. I think it is short in space but not in time!

1. Pages 87-9: 1 4 5 6 7 8 9

Notes: (1) Problem 1, p87, is a critical component in the proof of the Extreme Value Theorem. So please make sure you understand this result (and its proof) well! (2) For part (b), Problem 6, p88, you may want to refer to Problem 2 on Problem Set 5.

2. Pages 92-94: 1 2 4a 5 7 8 9

3. Let \((a_n)\) be a sequence in \(\mathbb{R}\). Prove that \((a_n)\) has a monotone (i.e. increasing or decreasing) subsequence. (Remember that a constant sequence is both increasing and decreasing. I’m pointing this out as a reminder that increasing doesn’t mean strictly increasing, etc.)

There are many ways to prove this result. One way is to use the result of extra credit problem 7 on the midterm exam as one case. Another is to consider these three cases. (These 3 cases were cleaned up (i.e., corrected) on Wednesday, Nov 6, 2013.)

Case 1: The sequence \((a_n)\) is not bounded above. In this case, it’s easy to produce an increasing subsequence.

Case 2: The sequence \((a_n)\) is not bounded below. In this case, it’s easy to produce a decreasing subsequence.

Case 3: The sequence is bounded. In this case, show that there is a monotone subsequence of \((a_n)\) converging to \(\lim \sup a_n\). Remember that the supremum of a set \(A\) need not be in the set \(A\). See Exercise 6, pages 54 and 55, which was part of problem set 4.

4. 10 points Extra Credit: Page 93, problem 10.