We can use exponential growth models to demonstrate how money in a savings account changes over time. Given an account with balance \( P(t) \) at time \( t \), and if interest is being accumulated by compounding continuously, then \( P(t) \) satisfies the differential equation \( \frac{dP}{dt} = kP \) where \( k \) is the annual interest rate. In this case, \( P(t) = P(0)e^{kt} \).

In real life it is rare to leave a bank account alone in this situation. We usually will add extra income into the account or will make withdrawals over time.

**Example 1.** A family is setting up a college fund for their eldest daughter. The family begins with $15,000 in the account, which earns 4% annually. They plan to deposit $7000 into the account annually until the daughter is 18. Assuming they don’t make any withdrawals, how much money will be in the account if the daughter is now 10 years old?
Example 2. Suppose you took out college loans totaling $75,000 with interest 7.5%. you have an online payment plan which continuously deducts money from your bank account at a rate that comes to $15,000 a year. How long will it take to pay off the loan?

Example 3. A drug is introduced intravenously at a rate of 0.5mg/hour. On a continuous basis 2% of the drug is removed from the blood and absorbed by the body. Write a differential equation satisfied by $y(t)$, the amount of drug in the patient, and determine the equilibrium amount.
Mixing problem: A problem where two or more substances are mixed together at various rates. The key here is that

\[ \frac{dy}{dt} = \text{rate in} - \text{rate out} \]

**Example 4.** A vat with 500 gallons of beer contains 4% alcohol (by volume). Beer with 6% alcohol is pumped into the vat at a rate of 5 gal/min and the mixture is pumped out at the same rate. What is the percentage of alcohol after an hour?
Example 5. A tank contains 1000 L of pure water. Brine that contains 0.05 kg of salt per liter of
water enters the tank at a rate of 5 L/min. Brine that contains 0.04 kg of salt per liter of water
enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the
tank at a rate of 15 L/min. How much salt is in the tank after $t$ minutes? after one hour?