The predator-prey system is a system that will arise several times throughout this course. In the natural world animals interact with each other, and those interactions often provide interesting systems to model.

Throughout this course we refer to our predators by the variable $F$ and our prey by the variable $R$ (you can think of foxes and rabbits, even though foxes rarely eat rabbits in the real world). We make the following assumptions:

- When no foxes are present, the population of rabbits will reproduce at a rate proportional to itself (and do so indefinitely).
- The foxes eat rabbits, and the rate at which the rabbits meet their demise is proportional to the rate of interaction between the two species.
- When no rabbits are present, the population of foxes will decline at a rate proportional to itself.
- The rate at which the foxes are born is proportional to the rate of interaction between the two species.

$\alpha$: the growth-rate coefficient for the rabbits.

$\beta$: the constant of proportionality that measures the fox-rabbit interactions in which the rabbit dies.

$\gamma$: the death-rate coefficient for the foxes.

$\delta$: the constant of proportionality that measures the fox-rabbit interactions that result in an increase in foxes.

We assume $\alpha, \beta, \gamma, \delta$ are all positive, and thus write the system as follows:

$$\frac{dR}{dt} = \alpha R - \beta RF$$

$$\frac{dF}{dt} = -\gamma F + \delta RF$$

This is a first-order system of ordinary differential equations. It is coupled because both $dR/dt$ and $dF/dt$ rely on both $R$ and $F$.

A solution to the system of equations is a pair of functions $R(t)$ and $F(t)$ that represent the populations of the rabbits and the foxes with respect to time. Because you must solve each differential equation simultaneously, we will not find exact general solutions, but will instead use qualitative and/or numerical methods to study the solutions.

An equilibrium solution is one in which both populations remain steady. That is, the number of foxes and rabbits neither increases nor decreases through the interactions.
Example 1. Consider the following two predator-prey models.

1) \[ \frac{dR}{dt} = 5R - 3RF \]
\[ \frac{dF}{dt} = -2F + \frac{1}{2}RF \]

2) \[ \frac{dR}{dt} = R - 8RF \]
\[ \frac{dF}{dt} = -2F + 6RF \]

1. In which system does the prey reproduce more quickly when there are no predators and equal numbers of prey?

2. In which system are the predators more successful at catching prey? That is, if the number of predators and prey is equal, in which system do the predators have the greater effect on the rate of change of the prey?

3. Which system requires more prey for the predators to achieve a given growth rate (assuming identical numbers of predators in both cases)?

4. For both systems, find the equilibrium solutions.

Example 2. Consider the following graphs of \( R(x) \) and \( F(x) \) for the solution of a given predator-prey system.

*This graphic is from www.boundless.com*

We state the following facts:

- The populations of the predators and the prey depend on each other.
- The population of the predators follows the population of the prey.
- When there are a lot of predators, then there are few prey. As the predators die from lack of food, the prey population increases, which in turn allows the system to support more predators.
- The solution to the system is periodic, but not trigonometric.