This is not meant to be an all-inclusive list of problems to practice, but will give you an idea of the expectations for the exam. No guarantees on whether I will post solutions, but we will discuss problems during our review period. I have included the numbers from the book so that you can look up the solutions and verify your work.

1. **(2.1: 9, 11, 13)** Consider the predator prey system

   \[ \frac{dR}{dt} = 2R - 1.2RF \]

   \[ \frac{dF}{dt} = -F + 0.9RF \]

   (a) How would you modify this system to include the effect of hunting of the prey at a rate of \( \alpha \) units of prey per unit of time?

   (b) Suppose the predators discover a second, unlimited source of food, but they still prefer to eat prey when they can catch them. How would you modify the system to include this assumption?

   (c) Suppose the predators migrate to an area if there are five times as many prey as predators in that area \( (R > 5F) \), and they move away if there are fewer than five times as many prey as predators. How would you modify the system to account for this?

2. Review some of the problems in Section 2.1 (like 7, 8, 16) and understand what the solution curves are saying.

3. **(2.2: 7)** Consider the second-order differential equation

   \[ \frac{d^2y}{dt^2} - y = 0. \]

   Convert it into a first-order system in terms of \( y \) and \( v \), where \( v = dy/dt \). Determine the vector field associated with the first-order system. Sketch enough vectors to get a sense of the geometric structure. Briefly describe the behavior of the solutions.

4. **(2.2: 19)** Consider the second-order differential equation

   \[ \frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y + y^3 = 0. \]

   Convert it into a first-order system in terms of \( y \) and \( v \), where \( v = dy/dt \). Determine the vector field associated with the first-order system. Find all equilibrium points. Briefly describe the behavior of the solutions.
5. Consider the harmonic oscillator equation
\[ \frac{d^2 y}{dt^2} + 3 \frac{dy}{dt} + 2y = 0. \]
Using the guess and check method from section 2.3, find two non-zero solutions that are not multiples of each other. For each solution, plot its solution curve in the \( yv \) plane, and its \( y(t) \) and \( v(t) \) graphs. Then solve the initial-value problem \( y(0) = 2, v(0) = -3 \).

6. Is the function \((x(t), y(t)) = (e^{-6t}, 2e^{-3t})\) a solution to the system of differential equations \( \frac{dx}{dt} = 2x - 2y^2 \) and \( \frac{dy}{dt} = -3y \)? Why?

7. Find a general solution to the system \( \frac{dx}{dt} = 2x \) and \( \frac{dy}{dt} = -3y \).

8. Consider the partially-decoupled system
\[ \frac{dx}{dt} = x + 2y + 1 \]
\[ \frac{dy}{dt} = 3y \]
Derive the general solution. Find the equilibrium points. Find the solution satisfying the initial value \((x_0, y_0) = (-1, 3)\).

9. Consider the system \( \frac{dY}{dt} = AY \). Check that the two functions are solutions to the systems; if they are not solutions then stop. Check the solutions are linearly independent. Solve the initial value problem.
   (a) \( A = \begin{pmatrix} -2 & -1 \\ 2 & -5 \end{pmatrix} \); \( Y_1(t) = (e^{-3t} - 2e^{-4t}, e^{-3t} - 4e^{-4t}) \), \( Y_2(t) = (2e^{-3t} + e^{-4t}, 2e^{-3t} + 2e^{-4t}) \); \( Y(0) = (2, 3) \).
   (b) \( A = \begin{pmatrix} -2 & -1 \\ 2 & -5 \end{pmatrix} \); \( Y_1(t) = (e^{-3t}, e^{-3t}) \), \( Y_2(t) = (e^{-4t}, 2e^{-4t}) \); \( Y(0) = (2, 3) \).

10. (3.2: 1-7 odd; 3.3: 1-5 odd) Compute the eigenvalues and eigenvectors. For each eigenvalue specify the straight-line solution and plot the \( x(t) \) and \( y(t) \) graphs. Then compute the general solution, if possible, and sketch the phase plane.
   (a) \( \frac{dY}{dt} = \begin{pmatrix} 3 & 2 \\ 0 & -2 \end{pmatrix} Y \)
   (c) \( \frac{dx}{dt} = -\frac{x}{2}, \frac{dy}{dt} = x - \frac{y}{2} \)
   (b) \( \frac{dY}{dt} = \begin{pmatrix} -5 & -2 \\ -1 & -4 \end{pmatrix} Y \)
   (d) \( \frac{dY}{dt} = \begin{pmatrix} 3 & 4 \\ 1 & 0 \end{pmatrix} Y \)

11. (3.2: 12; 3.3: 10) Solve the initial value problem
\[ \frac{dx}{dt} = 3x, \quad \frac{dy}{dt} = x - 2y \]
where the initial condition is \( (a) (1, 0) \) \( (b) (0, 1) \) \( (c) (2, 2) \).
Then sketch the solution curves in the phase plane and the \( x(t) \)- and \( y(t) \)- graphs.

12. Look at 3.3 #19-20.