Intro to Differential Equations
MATH 2070
Exponential and logarithmic functions review problems

Instructions: The symbol $\approx$ means “is approximately equal to.” For purposes of this homework set, assume that $\approx$ is actually equality. So, $\ln 2 \approx .7$ means here that $\ln 2 = 0.7$.

1. Given that $\ln 2 \approx .7$, $\ln 3 \approx 1.1$ and $\ln 5 \approx 1.6$, compute (or determine that it is not possible to compute from the information given) the following:

   a) $\ln 15$  
   b) $\ln 1.5$  
   c) $\ln 225$  
   d) $\ln 250$  
   e) $\ln 256$  
   f) $\log_5 27$  
   g) $\log_{27} 50$  
   h) $\log_6 5$  
   i) $\ln \frac{2}{15}$  
   j) $\ln 17$

2. Simplify exactly:

   (a) $e^{\ln 6}$  
   (b) $e^{-\ln 6}$  
   (c) $e^{2\ln 3}$  
   (d) $e^{3\ln 2}$  
   (e) $e^{\ln 11 - \ln 13}$

3. Solve exactly for $x$. (Your answers may involve logarithms).

   (a) $3^x = 5$. (Take ln of both sides)  
   (b) $3^{x+1} = 5^{2x}$.  
   (c) $7e^x - e^{2x} = 12$. (Since $e^{2x} = (e^x)^2$, you can produce a quadratic equation in the unknown $e^x$.)  
   (d) Shown in red is the graph of $y = 15e^x$ and in blue is the graph of $y = e^{2x} + 10$. The graphs cross at two points in the interval $-2 \leq x \leq 3$. Find the coordinates of these points.

![Graph of functions](image.png)
4. Calculus with exponential functions. Compute

(a) \( \frac{d}{dx} e^x \)
(b) \( \frac{d}{dx} e^{2x} \)
(c) \( \frac{d}{dx} (xe^{-2x}) \)
(d) \( \int e^x \, dx \)
(e) \( \int e^{2x} \, dx \)
(f) \( \int xe^{-2x} \, dx \) (Integrate by parts.)
(g) \( \int_0^1 e^x \, dx \)
(h) \( \int_0^1 e^{2x} \, dx \)
(i) \( \int_0^1 xe^{-2x} \, dx \)

5. Calculus with logarithmic functions. Compute

(a) \( \frac{d}{dx} \ln |x| \)
(b) \( \frac{d}{dx} \ln (x^2 + 1) \)
(c) \( \frac{d}{dx} (x \ln (x^2 + 1)) \)
(d) \( \int \frac{1}{x + 1} \, dx \)
(e) \( \int \frac{e^x}{e^x + 1} \, dx \)
(f) \( \int \frac{\cos (2x)}{\sin (2x) + 1} \, dx \)
(g) \( \int_0^1 \frac{1}{x + 1} \, dx \)
(h) \( \int_0^1 \frac{e^x}{e^x + 1} \, dx \)
(i) \( \int_{\pi/4}^{\pi/2} \frac{\cos (2x)}{\sin (2x) + 1} \, dx \)