MATH 3161 Final Exam Topic List

Your final exam will be next Tuesday, June 5th in our classroom (JGH 102) during class time (2:00-3:50 p.m.). You will have 110 minutes to work on the exam.

Exam Format: Closed book, notes, etc. No calculator. Cell phones will be off and may not be consulted during the exam.

Exam Length: Somewhere from 6-10 problems (depending on how many parts the problems have and how tough they are!) The exam is cumulative, however more of the exam will concentrate on material from AFTER the midterm. This is for two reasons: one is that you have already been tested once on pre-midterm material, and the other is that the post-midterm material necessitates knowledge of much of the pre-midterm material anyway. As with the midterm, there will be some questions (maybe 30-40% of the exam) which require no supporting work (e.g. recalling definitions or providing examples), and the remaining portion (maybe 60-70% of the exam) will be problems requiring proofs. The difficulty of these problems should be fairly close to that of your homework problems (though not the very hardest ones!) or some of the easier theorems proved in class (think “the sum of two convergent sequences is convergent,” not Bolzano-Weierstrass). Generally speaking, I am more interested in checking your ability to do several easier proofs rather than one gigantic difficult proof.

Showing work: In theory, you should prove everything either from basic building blocks of the real numbers (such as $0 \leq a \leq b$ and $0 \leq c \leq d$ imply $0 \leq ac \leq bd$) or clearly stated facts proved on the homework or in class (such as the fact that the union of two countable sets is countable). In reality though, try not to obsess about whether or not you’ve shown enough arithmetic work; in the end, the vast majority of the points will come from the correctness and clarity of your work and assertions, not from forgetting to show why the sum of two rational numbers is rational.

Sections covered in Abbott: 1.2, 1.3, 1.4 (didn’t do Existence of Square Roots), 1.5, 2.2, 2.3, 2.4 (didn’t do any topics involving series), 2.5, 2.6 (didn’t do Completeness Revisited), 3.2, 3.3, 3.4, 3.5, 4.2, 4.3, 4.4, 4.6 (didn’t do topics about monotone functions), 6.2.

Here is a list of definitions, concepts, and results that you should be familiar with.

- proof techniques: induction, double inclusion (to show that sets are equal), proof by contradiction
- correct usage of quantifiers
- absolute value
• triangle inequality (and corollaries such as $|x - y| \leq |x - z| + |y - z|$ and $||x| - |y|| \leq |x - y|$)
• upper bound, lower bound, sup, inf
• Axiom of Completeness
• Nested Interval Theorem
• Archimedean Principle (and the corollary that $\forall x \in \mathbb{R}$ with $x > 0$, $\exists n \in \mathbb{N}$ s.t. $\frac{1}{n} < x$)
• cardinality, what it means for two sets to have the same cardinality
• finite, countable and uncountable sets
• $\mathbb{R}$ is uncountable
• for any set $S$, the power set $\mathcal{P}(S)$ has a different cardinality than $S$
• know what it means for a sequence to converge, diverge, be bounded, be Cauchy
• know how to prove convergence of a sequence (with knowledge of the supposed limit) directly from the definition (i.e. exhibit $N$ for each $\epsilon > 0$)
• know how limits of convergent sequences respect arithmetic operations and $\leq$
• Monotone Convergence Theorem; know how to use the Monotone Convergence Theorem in conjunction with a proof that a sequence is monotone and bounded to prove that it converges
• Know what a subsequence is and basic facts about them (e.g. subsequences of a convergent sequence converge to the same limit as the original sequence)
• Bolzano-Weierstrass theorem
• A sequence is Cauchy if and only if it converges
• definitions of and basic properties of open and closed sets
• Limit points, isolated points, closure
• Know various definitions of compactness
• Nested Compact Set theorem
• Separated sets, connected sets
• Know what $F_\sigma$ and $G_\delta$ sets are
• Know Baire’s Theorem (that the intersection of countably many open dense sets is nonempty) and know why this implies that $\mathbb{Q}$ is NOT a $G_\delta$ set
• Limits of functions
• Know various definitions of continuity of a function at a point, know how to directly verify that a function is continuous with the $\epsilon$-$\delta$ definition
• Know how continuous functions can be combined to create more continuous functions (addition, multiplication, etc.)
• Know what makes a function uniformly continuous on a set; know that a continuous function on a compact set is automatically uniformly continuous
• Know that the set of discontinuities of any function $f : \mathbb{R} \to \mathbb{R}$ is a $F_\sigma$ set
• Know what it means for a sequence of functions to converge pointwise and/or uniformly on a set
• Know that the uniform limit of continuous functions is continuous