(1) If \( n \) lines are drawn in the plane \( (n \geq 3) \) so that no two of the lines are parallel, and no three of the lines meet in a single point, then show that among the regions that the lines partition the plane into, at least one is a triangle.

(2) A tromino is a rectangle whose dimensions are 1 cm by 3 cm. Let \( n \) be of the form \( 3k + 1 \) where \( k \) is a positive integer. Consider an \( n \times n \) square grid of little squares each 1 cm on a side. How many of the little squares have the following property: if deleted, the rest of the board can be perfectly covered by non-overlapping trominoes?

(3) Let \( \lfloor x \rfloor \) be the largest integer less than or equal to \( x \). Show that \( \lfloor (2 + \sqrt{3})^n \rfloor \) is odd for all positive integers \( n \).

(4) Let \( h(t) \) be a continuous function on the interval \([0, 1]\) such that \( h(0) = h(1) = 0 \). Show that there exists a real number \( x \in [0, \frac{2012}{2013}] \) such that \( h(x) = h(x + \frac{1}{2013}) \).

(5) Let \( n \) be a positive integer, and let \( a_1, \ldots, a_n \) be positive real numbers. Prove that \[
(a_1^{2013} + a_2^{2013} + \cdots + a_n^{2013})^{4013} \geq (a_1^{4013} + a_2^{4013} + \cdots + a_n^{4013})^{2013}.
\]

(6) Notice that the number 78 is a three-digit palindrome when written in base 5, since \( (78)_{10} = (303)_5 \); it is also a three-digit palindrome when written in base 7, since \( (78)_{10} = (141)_7 \). Prove that there are infinitely many positive numbers \( N \) that are three-digit palindromes to two different bases at the same time.

(7) Let \( T \) be a triangle with the following property: if \( T' \) is any triangle with the same perimeter and area as \( T \), then \( T' \) is actually congruent to \( T \). Show that \( T \) is equilateral.

(8) Which positive integers \( n \) have the property that there exist choices of the \( \pm \) signs for which
\[
\pm 1 \pm 2 \pm 3 \pm \cdots \pm (n - 1) \pm n = 0?
\]