Review

- Inconsistent heuristics can lead to an exponential number of re-expansions with A*
- Algorithm B interleaves Dijkstra searches with A* when inconsistencies are found
  - Reduces worst-case to $O(N^2)$

Worst case proof

- Worst case proof
  - $O(n^2)$ node expansions
- Each node can be opened with a maximum f-cost at most once
  - Between these openings, we are running Dijkstra’s algorithm
  - We will do at most n openings based on g-cost
  - $O(n^2)$
Solution - Mero 84 - B’

- Pathmax
  - When generating a node:
  - $h(n) = h(p) - c(n, p)$
  - $h(p) = \min(h(c) + c(c, p))$ over all children $c$

Algorithm B’

- Enhance B/A* with these steps:
  - 3(a) For each child $c_i$ of $n$, if $h(c_i) < h(n) - c(n, c_i)$ then set $h(c_i) \leftarrow h(n) - c(n, c_i)$
  - 3(b) Given child $c_i$ with min. cost $h(c_i) + c(c_i, n)$ set $h(n) \leftarrow \max(h(n), h(c_i) + c(c_i, n))$
- These are the pathmax rules
  - BPMX for undirected graphs applies both simultaneously / repeatedly

Pathmax rules

- Pathmax:
  - BPMX(1)

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Pathmax rules

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Pathmax rules

- Pathmax:

- BPMX(1)
So...how bad?

- In the worst case, even B’ can be $O(N^2)$
General Inconsistency Bounds

• Suppose $A^*$ performs $\varphi(N) > N$ expansions
  • Some node must be re-opened $(\varphi(N) - N)/N$ times
    • Pigeon-hole principle
  • If $\Delta$ is the minimum change in $h$-cost for a node
    • $h$-cost is at least $\Delta \cdot (\varphi(N) - N)/N$
    • Solution cost is also at least $\Delta \cdot (\varphi(N) - N)/N$
      • We never open a node with $f$-cost > solution

How do we get inconsistency in practice

• Special properties (duality)
• Compression
• Max of multiple heuristics
  • Too expensive to use all heuristics, so use random subset of heuristics

General Inconsistency Bounds

• Problem that grows exponentially
  • $\Delta = 1$, $N = b^d$, max heuristic is $h_{\text{max}}$
  • $h_{\text{max}} \geq \Delta \cdot (\varphi(N) - N)/N$
  • $h_{\text{max}} \geq (\varphi(N) - b^d)/b^d$
  • $\varphi(N) \leq h_{\text{max}} \cdot b^d + b^d = (h^* + 1) \cdot b^d = O(d \cdot N)$

General Inconsistency Bounds

• Problem that grows polynomially
  • $\Delta = 1$, $N = d^2$, max heuristic is $h_{\text{max}} / d$
  • $h_{\text{max}} = \Delta \cdot (\varphi(N) - N)/N$
  • $h_{\text{max}} = (\varphi(N) - d^2)/d^2$
  • $\varphi(N) = h_{\text{max}} \cdot d^2 + d^2 = O(d^3) = O(N^{1.5})$
What is good/bad inconsistency

- “Good” inconsistency
  - There are always good heuristics nearby
  - 1-step BPMX to ‘fix’ bad values
  - Improve the run-time distribution of h-values
- “Bad” inconsistency
  - Misleading values (worst path has lowest f-cost)
- Note: with no cycles, inconsistency isn’t a problem

BPMX in A*/IDA*

- BPMX is free in IDA*
- More expensive in A*
  - We don’t naturally backtrack through closed list
- Choice:
  - Backup as far as possible
    - O(N^2) cost or unbounded savings
  - Backup only k steps O(kN) cost