Lecture 3: Game Theory (light)

Example: Hop Step

- 121 board positions
  - Can be blocked or not blocked
- 2 players with 3 pieces each
  - Each in one of 121 positions
  - Lots of actions on pieces, so store piece locations
  - Also need to store parity (1/2) for each piece

Lecture Overview

- Game classification
  - Extensive form / Normal form
- Two-player / Multi-player
- Zero-sum / Non-zero sum
- Perfect information / Imperfect information
- Stochastic / Deterministic
- Nash equilibria
  - Equilibrium strategy / profile
- Nash equilibria in extensive form games
  - And/Or trees
  - Minimax

Administrative

- HW should be turned in
  - How was it?
  - Any brilliant ideas?
- Will run 500 game match between all players
  - Including self-play
  - Will run multiple matches in case performance changes drastically depending on random seeds
- Later in the class we will analyze how to play in detail
Normal Form game

- Also called a matrix game
- eg Rock, Paper, Scissors
- Simultaneous game
- Single-shot
  - Can be repeated

<table>
<thead>
<tr>
<th></th>
<th>Rock</th>
<th>Paper</th>
<th>Scissors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock</td>
<td>0, 0</td>
<td>-1, 1</td>
<td>1, -1</td>
</tr>
<tr>
<td>Paper</td>
<td>1, -1</td>
<td>0, 0</td>
<td>-1, 1</td>
</tr>
<tr>
<td>Scissors</td>
<td>-1, 1</td>
<td>1, -1</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

Extensive Form Game

- Sequential moves
- Could be simultaneous, but will show later
- Chess, Checkers
- Could be a graph
- can describe games both ways, but may have exponential blow up

Multi-Player Games

- Normal Form Example

<table>
<thead>
<tr>
<th></th>
<th>Rock</th>
<th>Paper</th>
<th>Scissors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock</td>
<td>-1, -1, 2</td>
<td>-1, 1, 0</td>
<td>1, -1, 0</td>
</tr>
<tr>
<td>Paper</td>
<td>1, -1, 0</td>
<td>-1, -1, 2</td>
<td>-1, 1, 0</td>
</tr>
<tr>
<td>Scissors</td>
<td>-1, 1, 0</td>
<td>1, -1, 0</td>
<td>-1, -1, 0</td>
</tr>
</tbody>
</table>

Multi-Player Games

- Extensive Form Example
### Zero-Sum Games
- Strictly competitive
- RPS / Nim

### Non-zero sum
- **Battle of the sexes**
  - Football: 3, 2
  - Opera: 0, 0

- **Prisoners Dilemma**
  - Coop.: -2, -2
  - Defect: -1, -5

### Perfect Information
- All players know all information available to other players
  - Chess
  - Checkers
  - Backgammon!
  - Dice rolls are public and \textit{a priori} unknown to both players

### Imperfect Information
- Both from simultaneous moves and hidden information
  - Extensive form game version of RPS
- Information set
Games with Chance (Non-determinism)

- Include chance nodes
  - Can introduce imperfect information
- Backgammon vs. Poker

Definitions

- $\sigma_i$ - a strategy for the $i$th player in the game
- $\sigma$ - a strategy profile
  - $\sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_n\}$
- $u_i(\sigma)$ - the utility of a strategy profile to the $i$th player
- $u(\sigma)$ - the utility for all players
  - $u(\sigma) = \{u_1(\sigma), u_2(\sigma), \ldots, u_n(\sigma)\}$

Nash Equilibrium

- A strategy profile, $\sigma$, is a Nash equilibrium if player $i$ cannot unilaterally increase $u_i(\sigma)$
  - $\forall \sigma_i' \ u_i(\sigma_i + \sigma_i') \leq u_i(\sigma)$
Nash Equilibrium Properties

- In two-player zero-sum games
  - All equilibria have the same payoff for all (both) players
  - All equilibria can be mixed without changing payoffs
  - Interchanged strategies are still N.E.
    - \( u(\sigma) = \{\sigma_1, \sigma_2\} \)
    - \( u'(\sigma) = \{\sigma'_1, \sigma'_2\} \)
    - \( u''(\sigma) = \{\sigma_1, \sigma'_2\} \)
    - \( u(\sigma) = u'(\sigma) = u''(\sigma) \)

Nash Equilibrium Properties

- In non-zero sum game or multi-player game
  - All equilibria do not have the same payoff for all players
  - Equilibria cannot be mixed without changing payoffs
  - Interchanged strategies are not N.E.