I'm just giving answers. No other information, such as solution method, is included here.

1. Solve 3 ways (1st order linear 2 ways, Laplace 1 way).
   (a) Solution: \( y(t) = 4 - 4e^{2t} \).
   (b) Solution: \( y(t) = -8te^{-t} \).

2. Solve the following IVPs. For parts (a), (b) and (c) determine how many times (if any) and when \( y(t) = 0 \) for \( t \geq 0 \).
   (a) Solution: \( y(t) = e^{-2t} + te^{-2t} \)
   (b) Solution: \( y(t) = (\cos t + \sin t) e^{-2t} \)
   (c) Solution: \( y(t) = e^{-t} \)
   (d) Solution: \( y(t) = 1 - t \)

3. Solution:
   \[
   y(t) = \frac{1}{3} (e^{-t} - e^{-4t}) + \frac{2}{3} u_1(t) (e^{-(t-1)} - e^{-4(t-1)}) + u_2(t) (e^{-(t-2)} - e^{-4(t-2)})
   \]
   Here’s the graph of the solution over the interval \([0, 6]\).
4. Solution:

\[ y(t) = e^{-2t} + 2u_1(t) e^{-2(t-1)} + 3u_2(t) e^{-2(t-2)} \]

Here’s the graph of the solution over the interval [0, 4].

5. Solutions

(a) \[ y(t) = \frac{1}{14(t^3 + 1)} (2t^7 + 7t^4 + 14t) \].

(b) \[ y(t) = t^4 + t \].

(c) \[ y(t) = 3(t^3 + 1)^{2/3} - 3 \].

6. Solution:

(a) If we let \( w(t) \) be the amount of salt in tank \( W \), etc., then the IVP is

\[
\begin{align*}
\frac{dw}{dt} &= -\frac{1}{8}w + x, \quad w(0) = 0 \\
\frac{dx}{dt} &= -x + \frac{2}{5}y + \frac{z}{20}, \quad x(0) = 15 \\
\frac{dy}{dt} &= -\frac{2}{5}y + \frac{1}{5}z, \quad y(0) = 0 \\
\frac{dz}{dt} &= \frac{1}{8}w - \frac{1}{4}z, \quad z(0) = 0
\end{align*}
\]

(b) Since there is twice as much liquid in tank \( Y \) than tank \( X \), twice as much liquid in \( Z \) than \( Y \), etc., this ratio will be preserved by the amount of salt in these tanks at equilibrium, giving

\[ x = 1, \quad y = 2, \quad z = 4, \quad w = 8 \]