1. For each of the following, either provide an example or use a theorem or fact from class to explain why such an example cannot exist. You DO NOT need to show work for your examples!

(a) A countable closed set

(b) A countable open set

(c) An open cover of \( \mathbb{R} \) with no finite subcover

(d) An open cover of \([0, 1]\) with no finite subcover

(e) A countable set with uncountable closure
2. Show that for any countable set $A$, there exist disjoint countable sets $A_1$ and $A_2$ so that $A = A_1 \cup A_2$. (Note: recall that empty or finite sets are NOT countable.)
3. Show that any convergent sequence is Cauchy.
4. Use the definition of an open set to show that if the sets $O_1$, $O_2$ are open, then $O_1 \cup O_2$ is also open.
5. Use only the definition of continuity to show that the function $f(x) = \frac{1}{x}$ is continuous at $x = 2$. 
6. Prove that if $O$ is open, then $O^c$ is closed.
7. Prove that the sequence of functions \( f_n(x) = \frac{x^2 + nx}{n} \) converges pointwise on \( \mathbb{R} \), but does not converge uniformly on \( \mathbb{R} \).
8. Prove that the finite union of compact sets is compact. (You may use without proof the fact that any finite set has a supremum and infimum for this problem, and you may use any definition of compactness that you like. Aside from this though, prove all facts you need.)
9. If \( f \) is a function which is integrable on intervals \([a, c]\) and \([c, b]\) for some real numbers \( a < c < b \), then show that \( \int_a^c f \, dx = \int_a^b f \, dx + \int_b^c f \, dx \).